Around + Around Solution:

Let's change the perspective of the problem first. Let's think of the merry-go-round as fixed and the home rotating around instead. We'll put this into a cart plane to analyze it better. The ant walks toward his home at any given time.

Let \((x(t), y(t))\) be the ant's position at time \(t\). Then since the merry-go-round is fixed with constant angular velocity we get \(\theta(t) = 2\pi t\).

\[
\frac{dx}{dt} = \cos(\theta(t)) \quad \text{and} \quad \frac{dy}{dt} = \sin(\theta(t))
\]

which yields

\[
x(t) = \frac{\sin(2\pi t)}{2\pi} + C \quad \text{and} \quad y(t) = \frac{-\cos(2\pi t)}{2\pi} + D.
\]

with initial conditions \(x(0) = 0 = y(0)\) we get

\[
x(t) = \frac{\sin(2\pi t)}{2\pi} \quad \text{and} \quad y(t) = \frac{1 - \cos(2\pi t)}{2\pi}
\]

which is a circle of radius \(\frac{1}{2\pi}\) centered at \((0, \frac{1}{2\pi})\). which makes sense because its circumference is 1 which is how far the ant will walk each minute.